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A Planck-scale axion and SU(2) Yang–Mills dynamics: present acceleration and the fate of the photon

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Abstract. From the time of CMB decoupling onwards we investigate cosmological evolution subject to a strongly interacting SU(2) gauge theory of Yang–Mills scale, $\Lambda \sim 10^{-4}$ eV (masquerading as the $\mathrm{U(1)_Y}$ factor of the SM at present). The viability of this postulate is discussed in view of cosmological and (astro-) particle physics bounds. The gauge theory is coupled to a spatially homogeneous and ultralight (Planck-scale) axion field. As first pointed out by Frieman et al., such an axion is a viable candidate for quintessence, i.e. dynamical dark energy, being associated with today's cosmological acceleration. A prediction of an upper limit for $\Delta t_{m_\gamma=0}$, the duration of the epoch stretching from the present to the point where the photon starts to be Meissner massive, is obtained: $\Delta t_{m_\gamma=0} \sim 2.2$ billion years.

1 Introduction

The possibility to interpret dark energy in terms of an ultralight pseudo-Nambu–Goldstone boson field is at the center of an exciting debate stretching over the last decade; see e.g. [1–6]. The idea is that an axion field ϕ, which is generated by Planckian physics, develops a small mass due to topological defects of a Yang–Mills theory. If the associated Yang–Mills scale is far below the Planck mass M_P, then ϕ's slow-roll dynamics at late time can mimic a small cosmological constant, one in agreement with present observations. Having the coherent field ϕ decay by increasingly efficient self-interactions at a late time, the associated very light pseudoscalar bosons interact with ordinary matter only very weakly and thus escape their detection in collider experiments. Because of a dynamically broken, global $\mathrm{U}(1)_A$ symmetry associated with the very existence of ϕ, the corresponding potential $V(\phi)$ is radiatively protected. Notice that $V(\phi)$ is generated by an explicit, anomaly-mediated breaking of $\mathrm{U}(1)_A$.

Up to small corrections, arising from multi-instantons effects, $V(\phi)$ has the following form [7, 8]:

$$V(\phi) = \mu^4 \left[1 - \cos\left(\frac{\phi}{F}\right)\right] . \quad (1)$$

Two mass scales enter in (1): the dynamical symmetry breaking scale F (the axion decay constant) and a scale μ associated with the explicit symmetry breaking. The scale μ roughly determines at what momentum scale the gauge theory providing the topological defects becomes strongly interacting. Recall that the potential (1) is an effective one, arising from a quantum anomaly of the $\mathrm{U}(1)_A$ symmetry, which is defined on integrated-out fermion fields. The anomaly becomes operative through topological defects of a Yang–Mills theory and is expressed by an additional, CP-violating contribution:

$$\mathcal{L}_{\phi-\mathrm{SU(2)}} = \frac{\phi}{32\pi^2 F} F^a_{\mu\nu} \widetilde{F}^{a,\mu\nu} . \quad (2)$$

Upon integrating over the topological sectors, one concludes that the parameter μ in (1) is comparable to the Yang–Mills scale Λ [7–9].

The mass m_ϕ of the field ϕ, as derived from (1) for the range $|\phi| \lesssim \frac{\pi}{2} F$, reads $m_\phi \simeq \mu^2/F$. Assuming $\phi \sim F \sim 10^{18}$ GeV, the value needed for μ to generate the present density of dark energy in the Universe (inferred from fits to SNe Ia luminosity distance–redshift data for $z < 1.7$ [10–12]) is $\mu \sim 10^{-3}$ eV [1]. By the closeness of μ's value to the MSW neutrino mass, a possible connection with neutrino physics was suggested in [1] (see also [6]).

In the present work we wish to propose a different axion-based scenario relating the (presently stabilized) temperature of the CMB, $T_\mathrm{CMB} = 2.35 \times 10^{-4}$ eV, with the present scale of dark energy $\sim 10^{-3}$ eV. Namely, we postulate that the gauge factor $\mathrm{U(1)_Y}$ of the standard model of particle physics (SM) is only an effective manifestation of a larger gauge group. According to [13] one is lead to consider $\mathrm{SU(2)} \supset \mathrm{U(1)_Y}$[1] (henceforth referred to as $\mathrm{SU(2)_{CMB}}$)

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[1] For the present discussion we disregard the fact that in the SM the unbroken generator, corresponding to $\mathrm{U(1)_{em}}$, is a linear combination, parametrized by the Weinberg angle, of the

as a viable candidate for such an enlargement of the SM's gauge symmetry. In spite of the fact that such a postulate is rather unconventional, we nevertheless feel that a fruitful approach to the dark-energy problem needs novel Ansätze. In a slightly different context a QCD-like force of scale $\sim 10^{-3}$ eV was also discussed in [4].

We intend to explore some consequences of the postulate $\mathrm{SU(2)_{CMB}} \overset{\text{today}}{=} \mathrm{U(1)_Y}$ in connection with axion physics. The observation of a massless and unscreened photon strongly constrains the region in the phase diagram of the SU(2) Yang–Mills theory corresponding to the present state of the Universe [13, 14]. As a consequence, the Yang–Mills scale Λ_{CMB} is determined and found to be comparable to T_{CMB}: $\Lambda_{\mathrm{CMB}} \sim 10^{-4}$ eV.

The fact that T_{CMB} is comparable to the Yang–Mills scale of a theory with gauge group $\mathrm{SU(2)_{CMB}}$ (containing the $\mathrm{U(1)}_Y$ factor of the SM as a subgroup), which in connection with a Planck-scale axion field explains the present density of dark energy, does by itself not constitute a proof for the existence of $\mathrm{SU(2)_{CMB}}$ in Nature. For this setup to be convincing it ought to make independent and experimentally verified pre- and postdictions, such as a dynamical account of the large-angle features of CMB maps induced by the nonabelian fluctuations of $\mathrm{SU(2)_{CMB}}$. In this sense, the present paper is the very first stage in a long-term program exploring the implications of $\mathrm{SU(2)_{CMB}} \overset{\text{today}}{=} \mathrm{U(1)_Y}$ [15]. The authors are well aware of the fact that this program may lead to the falsification of the postulate $\mathrm{SU(2)_{CMB}} \overset{\text{today}}{=} \mathrm{U(1)_Y}$. Mounting evidence for its correctness is, however, provided by two-loop calculations of thermodynamical quantities in the deconfining phase of SU(2) Yang–Mills theory [14, 16, 17], making a further pursuit of this program worthwhile.

For the reader's convenience, let us put the results of [13] into perspective with other approaches to Yang–Mills thermodynamics. First of all, it is important to note that [13] considers the case of pure thermodynamics only; that is, the absence of external (static or dynamic) sources that would upset the spatial homogeneity of the system. As a consequence, the approach in [13] has nothing to say about the spatial string tension in the deconfining phase. The spatial string tension, introducing a distance scale R into the system, can, however, be easily extracted in lattice simulations of the spatial Wilson loop [18–23]. It is conceivable that static sources can be treated adiabatically, based on the approach of [13], by assuming a position dependence of temperature, but this is the subject of future research. In the absence of sources, a situation that is of relevance for physics on cosmological length scales, [13, 24] give a detailed and reliable account of SU(2) and SU(3) Yang–Mills thermodynamics. Lattice simulations for SU(3), using the differential method [25, 26], which is adapted to finite lattice sizes, yield quantitative agreement with the result for the entropy density (an infrared safe quantity) obtained in [13]. The results for the pressure and the energy density (infrared sensitive) agree qualitatively with those obtained in [25, 26]. That is, in contrast to the integral method, which assumes the infinite-volume limit on a finite-size lattice, the pressure is negative a little above the critical temperature in the deconfining phase, and there is a power-like, fast approach to the Stefan–Boltzmann limit for $T \to \infty$. Both phenomena are observed in the approach of [13], but numerically the results differ in the vicinity of the phase boundary.[2] By virtue of the trace anomaly the gluon condensate, up to a factor weakly depending on temperature (β-function over fundamental coupling g), coincides with the trace of the energy-momentum tensor $\rho - 3P$. Neglecting the masses and interactions of the excitations, which is an excellent approximation at high temperatures as far as the excitation's equation of state is concerned, we have $\rho - 3P \propto T$. In [27], the temperature dependence of the SU(2) gluon condensate was investigated on the lattice, and, indeed a linear rise of the gluon condensate with temperature was observed. Notice the conceptual and technical differences of [13] to the hard-thermal-loop (HTL) approach [28]. The latter derives a nonlocal theory for interacting soft and ultrasoft modes. While the HTL approach, in a highly impressive way technically, integrates perturbative ultraviolet fluctuations into effective vertices, it cannot shed light on the stabilization of the infrared physics associated with nonperturbative fluctuations residing in the magnetic sector of the theory. The derivation of the phase $\phi/|\phi|$ in [13, 29], however, invokes these nonperturbative, magnetic correlations. An impressive machinery has been developed within the renormalization-group flow approach to Yang–Mills thermodynamics both in the imaginary- [30, 31] and the real-time approach [32, 33], the latter reference making important observations concerning the (nonperturbative) temperature dependence of the thermal gluon mass. Interesting results for the nonperturbative temperature dependence of the fundamental gauge coupling in quantum chromodynamics were obtained in [34]. However, to the best of the authors' knowledge, no decisive nonperturbative calculation (fully considering the magnetic sector) of the thermodynamical pressure or related quantities has yet been performed within this approach.

The paper is organized as follows: First, we briefly recall some basic nonperturbative results obtained in [13] for SU(2) Yang–Mills thermodynamics. Then we discuss the viability of our scenario when confronting it with particle physics experiments and cosmological observations. Subsequently, we consider for a spatially flat Universe the evolution of the cosmological scale factor $a = a(t)$ and of the axion field $\phi = \phi(t)$ from the time of decoupling t_{dec}

diagonal $\mathrm{SU(2)_W}$ generator and the $\mathrm{U(1)_Y}$ generator in unitary gauge, since we are not concerned with the interactions of the photon with electrically charged leptons and/or hadrons that would make this mixing operative. Our investigation of cosmology below sets in at the point where the CMB decoupling takes place. The issue is, however, re-addressed in Sect. 3.

[2] We believe that this is an artifact of the finite lattice volume, which close to the phase boundary affects the long-range correlations present in the ground state. Hence we dismiss the widely used argument that the imprecise knowledge of the lattice β-function would be the cause of the *apparent* problems with the differential method.

(corresponding to $z_{\rm dec} = 1089$) up to the present. We then investigate the future evolution of the Universe up to the point when the transition between the deconfining and the preconfining phase of $SU(2)_{\rm CMB}$ will take place and the photon will acquire a Meissner mass. Finally, we present our conclusions and an outlook on future research.

2 SU(2) Yang–Mills thermodynamics

In [13] a nonperturbative approach for SU(2)/SU(3) Yang–Mills thermodynamics is developed. For the sake of brevity we recall only some of the results relevant for the present study. Analytical expressions are reported in the appendix.

2.1 Deconfining (electric) phase

It is shown in [13, 29] that at high T an inert adjoint Higgs field φ emerges upon spatial coarse-graining over topological defects. The modulus of the Higgs field is (nonperturbatively) temperature dependent with $|\varphi_{\rm E}| = \sqrt{\Lambda_{\rm E}^3/2\pi T}$ (here $\Lambda_{\rm E}$ denotes the Yang–Mills scale as defined in the deconfining phase [13]); furthermore, the field $\varphi_{\rm E}$ induces dynamical gauge symmetry breaking: $SU(2) \to U(1)$; two of the three gauge bosons become massive (denoted by $V^\pm$), while the third one remains massless (denoted by γ). Massive excitations $V^\pm$ are very weakly interacting thermal quasi-particles; their mass depends on temperature as follows:

$$m_{V^\pm} = 2e(T)\,|\varphi_{\rm E}|\;, \qquad (3)$$

where $e(T)$ is an *effective* temperature-dependent gauge coupling, as plotted in Fig. 1. The gauge coupling $e(T)$ diverges logarithmically at $T_{\rm c,E}$ and has a plateau value of $e \simeq 8.89$ for $T \gg T_{\rm c,E}$. For $T \searrow T_{\rm c,E}$, the $V^\pm$ bosons acquire an infinite mass, $m_{V^+} = m_{V^-} \propto -\log(T_{\rm E} - T_{\rm c,E})$. Thus they do no longer (weakly) screen the propagation of the massless excitation γ.

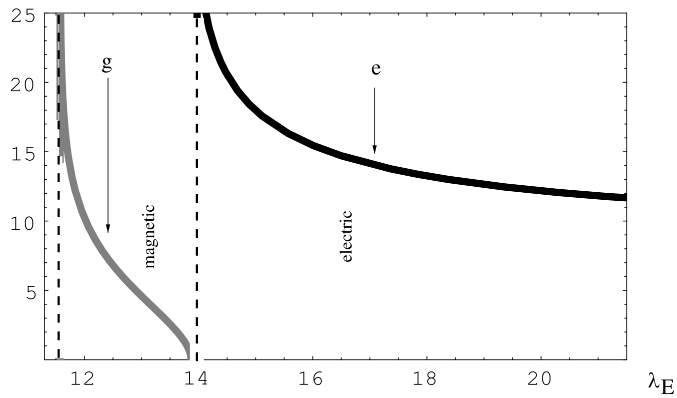

Fig. 1. Temperature evolution of the effective (electric and magnetic) couplings

Plots for the energy density and for the pressure as functions of the dimensionless temperature $\lambda_{\rm E} = \frac{2\pi T}{\Lambda_{\rm E}}$ are shown in Fig. 2.

In Fig. 2a a jump in the energy density at the critical value $\lambda_{\rm c,E} = 13.87$ is seen. This signals a transition between the deconfining (also called electric, $T \geq T_{\rm c,E}$) and the preconfining (also called magnetic, $T_{\rm c,M} \leq T < T_{\rm c,E}$) phases. Notice that, for high T, the Stefan–Boltzmann limit is approached in a power-like fashion.

2.2 Preconfining (magnetic) phase

In the preconfining phase the dual gauge symmetry U(1) is dynamically broken by a magnetic monopole condensate, which, after spatial coarse-graining, is described by an inert complex scalar field $\varphi_{\rm M}$ with $|\varphi_{\rm M}| = \sqrt{\Lambda_{\rm M}^3/2\pi T}$. The dual gauge excitation γ now acquires a Meissner mass:

$$m_\gamma = g(T)\,|\varphi_{\rm M}|\;, \qquad (4)$$

where $g(T)$ is the effective coupling in the magnetic phase as shown in Fig. 1. The latter vanishes for $T \nearrow T_{\rm c,E}$, and it diverges logarithmically for $T \searrow T_{\rm c,M}$, where $T_{\rm c,M} \sim 0.83 T_{\rm c,E}$ in the SU(2) case.

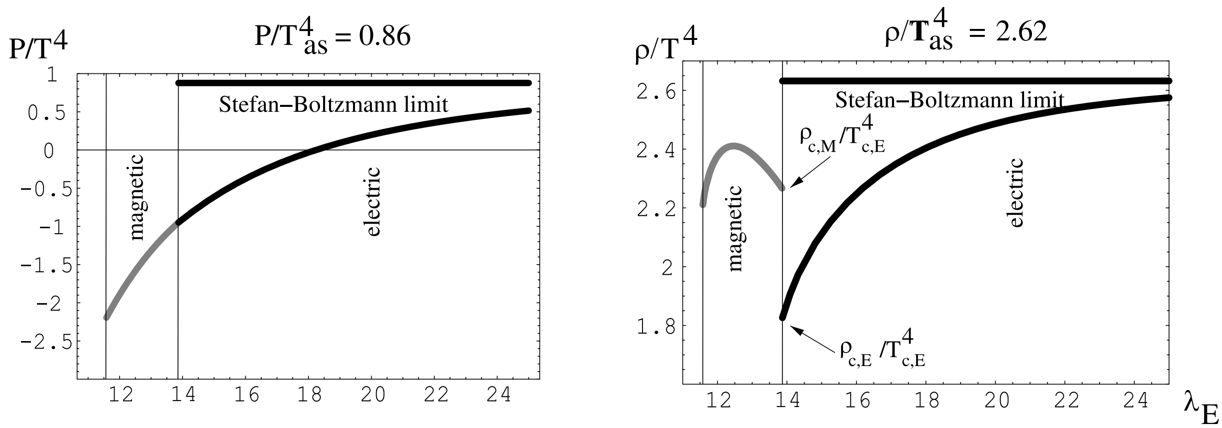

Fig. 2. Ratios of pressure (*left panel*) and energy density (*right panel*) to T^4 of an SU(2) Yang–Mills theory within the deconfining (electric) and preconfining (magnetic) phases. The *vertical lines* denote phase boundaries

For $T \nearrow T_{\mathrm{c,E}}$, a jump in the number of polarizations ($3 \to 2$) takes place, thus explaining the discontinuity $\rho_{\mathrm{c,M}} \to \rho_{\mathrm{c,E}}$ in the energy density; see Fig. 2. Due to the dominance of the ground state the pressure is negative for $T \sim T_{\mathrm{c,E}}$ (for a microscopic explanation of this macroscopically stabilized situation, see [13]).

For $T \searrow T_{\mathrm{c,M}}$, the excitation γ becomes infinitely massive, thus signaling a second phase transition that is of the Hagedorn type.

2.3 Confining (center) phase

For temperatures below $T_{\mathrm{c,M}}$ the system is in its confining phase: fundamental test-charges and gauge modes are confined and decoupled, respectively. The excitations are (single and self-intersecting) fermionic center-vortex loops [13].

When confronting these results with the postulate $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$ the reader may be puzzled by the existence of two massive excitations in addition to the photon for $T > T_{\mathrm{CMB}}$; however, as shown in [13] and explained in more detail in Sect. 3, the interaction between γ and $V^{\pm}$ is tiny, because the off-shellness of admissible quantum fluctuations is strongly constrained by the applied spatial coarse-graining. At $T = T_{\mathrm{c,E}} = T_{\mathrm{CMB}}$, where the $V^{\pm}$ disappear from the thermal spectrum, γ is exactly noninteracting. It is an experimental fact that today's on-shell photons are massless on the scale of $T_{\mathrm{CMB}} = 2.35 \times 10^{-4}\,\mathrm{eV}$ ($m_\gamma < 10^{-13}\,\mathrm{eV}$ [51]), and they are unscreened. As a consequence, the postulated $\mathrm{SU(2)_{CMB}}$ dynamics necessarily is characterized by a temperature $T_{\mathrm{c,E}} \sim T_{\mathrm{CMB}}$ today: $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$. This entails $\Lambda_{\mathrm{CMB}} \sim 1.065 \times 10^{-4}\,\mathrm{eV}$ and a present $\mathrm{SU(2)_{CMB}}$ ground-state pressure $P^{\mathrm{g.s.}} \sim -(2.44 \times 10^{-4}\,\mathrm{eV})^4$. Before discussing the phenomenological and cosmological viability of $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$, two comments are in order.

(i) For $T \searrow T_{\mathrm{c,E}}$, the $\mathrm{SU(2)_{CMB}}$ system will not immediately jump to the preconfining phase because of the discontinuity in the energy density; see Fig. 2 and the corresponding evaluation in the appendix. Rather, it remains in a supercooled state until a temperature $T_* \in (T_{\mathrm{c,M}}, T_{\mathrm{c,E}})$ is reached, at which a restructuring of the ground state (interacting calorons $\to$ interacting, massless monopoles [13]) does not cost any energy. For a detailed discussion of this situation, see Sect. 5.

(ii) If $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$ strictly holds, then it was not always so: for $T > T_{\mathrm{CMB}}$, photons did interact with the massive partners $V^{\pm}$. Nonabelian effects peak at $T \sim 3T_{\mathrm{c,E}}$ and are visible on the level of 10^{-3} in the relative deviation from the ideal photon-gas pressure [14, 16, 17]. This represents a crucial test for our basic postulate. We suspect that the effect generates a dynamical contribution to the dipole of the CMB temperature map in addition to a component induced by the relativistic Doppler effect [35]. A more quantitative analysis of this assertion is beyond the scope of the present paper but is planned as a next step.

3 But is it viable?

To see that the suggestion $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$ is viable when confronted with observational facts in cosmology (nucleosynthesis) and (astro-) particle physics bounds on neutral and charged-current interactions, we need to consider the following points.

(a) First, we should consider big bang nucleosynthesis (BBN) bounds on the number of relativistic degrees of freedom at $T \sim 1\,\mathrm{MeV}$ in the SM. To resolve an apparent contradiction with $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$, we discuss the underlying gauge dynamics of the weak sector, which now is based on pure SU(2) Yang–Mills dynamics. The latter entails results such as a Higgs-particle free and stepwise mechanism for electroweak symmetry breaking and the dynamical emergence of the lepton families.

(b) Next, we have constraints on the γ–$V^{\pm}$ interaction for $T > T_{\mathrm{CMB}}$.

(c) Third, we should consider charged currents. This is of relevance for the discussion of supernova cooling.

(d) Finally, there is a γ interaction with charged leptons and neutral currents (particle–wave duality of the photon).

(a) Numbers of relativistic degrees of freedom. Relying on the observed primordial ^{4}He and D abundances and on a baryon to photon number density ratio η ranging in $4.9 \times 10^{10} \le \eta \le 7.1 \times 10^{10}$, SM-based nucleosynthesis predicts that the number of relativistic degrees of freedom g_* at the freeze-out temperature $T_{\mathrm{fr}} \sim 1\,\mathrm{MeV}$ is given by

$$g_* = 5.5 + \frac{7}{4} N_\nu \,, \tag{5}$$

with $1.8 \le N_\nu \le 4.5$ [36]. This prediction relies on the following argument: the neutron to proton fraction n/p at freeze-out is given by $n/p = \exp[-Q/T_{\mathrm{fr}}] \sim 1/6$, where $Q = 1.293\,\mathrm{MeV}$ is the neutron–proton mass difference, and one has

$$T_{\mathrm{fr}} \sim \left(\frac{g_* G_{\mathrm{N}}}{G_{\mathrm{F}}^4} \right)^{1/6} . \tag{6}$$

In (6) G_{N} denotes the Newton constant, and

$$G_{\mathrm{F}} = \pi \frac{\alpha_{\mathrm{w}}}{\sqrt{2} m_W^2} \sim 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2} \tag{7}$$

is the Fermi coupling at zero temperature. To use the zero-temperature value of G_{F} at $T_{\mathrm{fr}} = 1\,\mathrm{MeV}$, as it is done in (6), is justified by the large 'electroweak scale' $v = 247\,\mathrm{GeV}$, the vacuum expectation of the fundamentally charged Higgs field in the SM. $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=} \mathrm{U(1)_Y}$ tells us that there are effectively six relativistic degrees of freedom at $T_{\mathrm{fr}} = 1\,\mathrm{MeV}$ in addition to the situation described by the SM: a result that clearly exceeds the above cited upper bound for N_ν. But does this falsify our postulate $\mathrm{SU(2)_{CMB}} \stackrel{\mathrm{today}}{=}$

$U(1)_Y$, or is there new physics associated with the thermalization of the weak sector of the SM? In what follows, we will argue that the approach to Yang–Mills theory sketched in Sect. 2 should also be applied to the electroweak group $SU(2)_W$ (which we refer to as $SU(2)_e$ where e stands for 'electron'; see below). In [13] we have discussed why and how the assignment $SU(2)_W = SU(2)_e$ (the associated Yang–Mills scale is $\Lambda_e \sim m_e \sim 0.5$ MeV) works to generate a triplet of intermediary massive vector bosons: $W^\pm$ decouples at the deconfining–preconfining phase boundary $T = T_{c,E}$ (second-order-like transition), while Z_0 decouples at the boundary between the preconfining and confining phase $T = T_{c,M}$ (Hagedorn transition). Thus the weak symmetry is broken in a stepwise fashion (notice that $\Lambda_e \sim m_e \sim T_{c,M} \sim 0.835 T_{c,E}$). Moreover, the first lepton family (e, ν_e) emerges in the confining phase of $SU(2)_e$ (single and self-intersecting center-vortex loop; all higher-charge states are unstable). The effective V–A structure of the weak currents likely emerges as a consequence of the departure from (local) thermal equilibrium close to the Hagedorn transition, causing the CP-violating Planck-scale axion to fluctuate. The mass of the self-intersecting center-vortex loop m_e (mass of the electron) is roughly equal to the scale Λ_e. Furthermore, the single center-vortex loop emerges as a Majorana particle [13] in agreement with experiment [37]. As another consequence of $SU(2)_W = SU(2)_e$, the hierarchy $g_{dec}^{-1} \equiv g^{-1}(T_{c,M}) \sim \frac{m_e}{m_{Z_0}} \sim 10^{-5} \sim \frac{m_{\nu_e}}{m_e}$ is *not* explained by the large value of v, the Higgs expectation, a moderate value of the gauge coupling, and a B–L forbidden neutrino mass, but by the logarithmic pole $g \sim -\log(T - T_{c,M})$ of the magnetic coupling [13]. Thus the high-energy scale associated with the Higgs sector of the SM turns out to emerge in terms of *pure* but nonperturbative Yang–Mills physics.

Now at $T_{fr} = 1$ MeV, $SU(2)_e$ is in its deconfining phase. As a consequence, Z_0 is massless at T_{fr}, and the mass of $W^\pm$ is substantially reduced compared to its decoupling value [13]. The latter, in turn, is responsible for the zero-temperature value of G_F; see (7)). This, however, implies that the G_F in (6) is substantially enhanced compared to its zero-temperature value. As a consequence, a larger number g_* than obtained in the SM calculation follows. To determine g_* from the observed primordial abundances of the light elements one needs to perform the detailed simulation invoking the above dynamics. This is beyond the scope of the present paper.[3]

To summarize, as far as the first lepton family and its interactions is concerned, the electroweak sector of the SM is likely described by pure $SU(2)_{CMB} \times SU(2)_e$ dynamics where the gauge modes of $SU(2)_e$ are very massive for $T < m_e$. In a similar way, the doublet (ν_μ, μ) corresponds to the (stable) excitations of an $SU(2)_\mu$ pure gauge theory with $\Lambda_\mu \sim m_\mu \sim 200\, m_e$ (and also to three very massive intermediary gauge modes $\Omega^\pm, \Omega_0$ not yet detected). The corresponding group structure is a direct product of SU(2) Yang–Mills factors responsible for the existence of leptons and their interactions:

$$SU(2)_{CMB} \times SU(2)_e (= SU(2)_W) \times SU(2)_\mu \times \cdots \quad (8)$$

with nontrivial mixing.[4] In view of the above scenario the electroweak sector of the SM emerges as a low-energy effective theory being valid for momentum transfers ranging from zero up to values not much larger than $m_{Z_0} \sim 91$ GeV for an isolated vertex or for temperatures not exceeding $m_e \sim 0.5$ MeV.

(b) γ–$V^\pm$ interaction. Let us now discuss why the γ excitation of $SU(2)_{CMB}$ does practically not radiate off or create $V^\pm$ pairs, why γ is not created by the annihilation thereof, and why there is practically no scattering of γ off of $V^\pm$.

In Sect. 2 we introduced the Higgs field φ_E, describing the BPS saturated part of the ground-state dynamics in the deconfining phase. In a physical (unitary-Coulomb) gauge, quantum fluctuations of nontrivial and trivial topology are integrated out down to a resolution $|\varphi_E|$ in the effective theory. As a consequence, one has for the off-shellness of residual quantum fluctuations

$$|p^2 - m^2| \leq |\varphi_E|^2 \quad (9)$$

for the momentum transfer in a four-vertex (deconfining phase)[5]

$$|(p+q)^2| \leq |\varphi_E|^2 \,. \quad (10)$$

Notice that the constraints (9) and (10) do not apply close to the Hagedorn transition at $T_{c,M}$; in fact, in the critical region of preconfining phase $\leftrightarrow$ confining phase, thermal equilibrium breaks down: the 't Hooft loop undergoes rapid and local phase changes violating spatial homogeneity [13]. Thus a limit on the maximal resolution, as it emerges in the thermalized situation ($T \ll T_{c,M}$ and $T > T_{c,M}$), no longer exists close to the Hagedorn transition.

If it were not for the constraints (9) and (10), the effective theory would be strongly interacting; recall that $e \sim 8.89$ for $T \gg T_{c,E}$), and the postulate $SU(2)_{CMB} \overset{\text{today}}{=} U(1)_Y$ surely would not be viable. As $|\varphi_E|$ decays like $|\varphi_E| = \sqrt{\Lambda_E^3 / 2\pi T}$, the constraints (9) and (10) become

[3] Ideal testing grounds for the postulate $SU(2)_W = SU(2)_e$ subject to the nonperturbative approach of [13] are the physics of the solar core and the central plasma region of a state-of-the-art tokamak: the constant flux of 10^{43} protons per year [38], the solar wind, badly violates electric charge conservation, which, according to the SM, should hold at the temperatures prevailing in the solar core. Moreover, the onset of the Hagedorn transition from the confining to the preconfining phase, which violates spatial homogeneity, possibly is detected by microturbulences within the magnetically confined plasma of a tokamak at a central temperature ~ 40 keV $\sim 1/10\, m_e$ [39]. For other indications that $SU(2)_W = SU(2)_e$ is in agreement with experiment, see [13] and related references therein.

[4] As for the mixing a tacit assumption is that the above gauge symmetry is a remnant of a breaking $SU(N \gg 1) \to SU(2)_{CMB} \times SU(2)_e (= SU(2)_W) \times SU(2)_\mu \times \cdots$ at energies not too far below the Planck mass.

[5] We do not distinguish s, t, and u channels here. See, however, [24].

tighter and tighter with increasing temperature T. For example, the modulus of the ratio of two-loop corrections to the one-loop result for the thermodynamical pressure in the deconfining phase rapidly approaches 4×10^{-4} for $T\gg T_{\rm c,E}$ and has a peak of $\sim10^{-2}$ at $T\sim3\,T_{\rm c,E}$ [14, 16, 17]. Thus, the tiny interactions at high temperature can be absorbed into a tiny shift of the temperature in a free-gas expression of the pressure for massless gauge modes; see the appendix.

There is, indeed, a regime $T\stackrel{\sim}{>}T_{\rm CMB}$, where a small fraction, $\sim10^{-3}$, of the γ excitations is converted into $V^\pm$ pairs and vice versa, or where there is very mild scattering of γ off of $V^\pm$. While this is of (computable) relevance for CMB physics at redshift $z\sim2$–10 or so [14, 16, 17], there is no measurable effect in collider experiments, atomic physics, and astrophysical systems.[6]

(c) Charged currents. By virtue of the gauge structure proposed in (8), one would expect the presence of additional charged-current interactions. Let us discuss the prototype of such an interaction in the SM: the decay $\mu^\pm\to e^\pm+\bar\nu_e+\nu_\mu$. The immediate question is why this decay, which is mediated by an intermediary $W^\pm$ in the SM, is not enhanced by $V^\pm$ mediation through a nontrivial mixing of $W^\pm$ and $V^\pm$. On the scale of $m_\mu\sim10^{12}\times T_{\rm CMB}$ the $V^\pm$ modes are practically massless, even if we assume a decoupling mass $\sim10^5\times T_{\rm CMB}$ in analogy to the experimentally accessible case of $\mathrm{SU}(2)_e$. We have $|\varphi_{\rm E}|\sim T_{\rm CMB}$. Now the momentum transfer in the decay is comparable to m_μ, and thus the condition in (9) would be badly violated for a $V^\pm$ intermediary fluctuation. Thus, mediation of the decay by $V^\pm$ is strictly forbidden. Mediation by an $W^\pm$ intermediary fluctuation is, however, allowed through the mixing $\mathrm{SU}(2)_{\rm CMB}\leftrightarrow\mathrm{SU}(2)_e$, although this particle is also far away from its mass shell: in contrast to $V^\pm, W^\pm$ is virtually excited *across* the Hagedorn phase boundary of $\mathrm{SU}(2)_e$, where the constraints in (9) and (10) do not apply. The exclusion of $V^\pm$ mediation in $\mu^\pm$ decay represents all other charged-current processes with a momentum transfer exceeding $m_V^\pm\sim10\,\mathrm{eV}$. The discussion in *(b)* and *(c)* is important to not contradict the neutrino luminosity measured in the SN 1987A cooling pulse[7], see for example [40], and to be below the imposed experimental cuts for missing momenta in collider experiments.

Let us remark that on the level of (8), that is, resolving the local SM vertex, the decaying soliton $\mu^\pm$ first must (nonlocally) couple to the soliton ν_μ and to $\Omega^\pm$, which subsequently rotates into $W^\pm$. The latter (nonlocally) couples to the solitons ν_e and $e^\pm$. On the effective level of the SM only a $W^\pm$ mediation appears with *local* coupling to $\mu^\pm$, ν_μ, ν_e, and $e^\pm$, which are all treated as point particles. The SM vertex for charged currents follows from an (effective) $\mathrm{SU}(2)_{\rm W}$ gauge principle. Obviously, its derivation in terms of complex dynamics governed by (8) is an extremely complicated task; see for example [44]. It may or may not be accomplished in the future. Seen in this light, the SM is an effective (and ingenious) quantum field theoretic setup describing the *interactions* between *postulated* point particles of a given (effective) gauge charge. The gauge structure proposed in (8) facilitates a deeper understanding of gauge symmetry breaking, of the ground-state structure of our Universe, of zero-temperature particle properties such as the classical magnetic moment and the classical self-energy (mass of the electron, ...), and of the high-temperature behavior of particle physics.

(d) γ interaction with charged leptons and neutral currents. There is an important difference with the charged-current case. Namely, *electrically* charged (with respect to the defining fields of SU(2)), far-off-shell $W^\pm$ bosons *induce magnetically* charged monopoles[8], represented by the self-intersection region of a center-vortex loop, and vice versa while there is a *coupling* of the dual gauge boson to the *magnetically* charged monopoles. By virtue of (9), the excitation γ of $\mathrm{SU}(2)_{\rm CMB}$ is only allowed a maximal off-shellness, comparable to $T_{\rm CMB}$. This means that by itself it cannot mediate electromagnetic interactions with momentum transfer on atomic physics scales or on even higher intrinsic scales. It could not do so anyway in the absence of mixing between the propagating excitations of $\mathrm{SU}(2)_{\rm CMB}$ and $\mathrm{SU}(2)_e$, $\mathrm{SU}(2)_\mu$, $\cdots$, because the γ excitation simply would not 'see' the charged leptons. Since such a (universal) mixing exists, a sufficiently off-shell γ mode never is emitted by a charged lepton, but rather the associated dual gauge mode Z_0, Ω_0, $\cdots$ of $\mathrm{SU}(2)_e$, $\mathrm{SU}(2)_\mu$, $\cdots$ In contrast to γ, the latter are allowed to be off-shell across their respective Hagedorn boundaries. The Z_0, Ω_0, $\cdots$, in turn, couple to their charged leptons with a large magnetic coupling $g_{\rm dec}\sim10^5$. In contrast to the charged-current process associated with a parametric suppression $\frac{p^2}{m_W^2}$ (p being the transferred momentum) the coupling of Z_0, Ω_0, $\cdots$ to the associated charged leptons leads to an enhancement $\frac{(g_{\rm dec}p)^2}{m_{Z_0}^2}$, explaining why electromagnetic interactions are so much stronger than weak interactions. Again, the local SM vertex between a charged lepton and a massless photon, determined by a universal (effective) U(1) gauge symmetry, is an extremely efficient and successful description and is very hard to derive from the

[6] Radiowave propagation occurs on a thermalized CMB background with decoupled $V^\pm$ today. That is, the present Universe's thermalized ground state does not allow for the creation of these particles as intermediary fluctuations, left alone their on-shell propagation. Exceptional astrophysical systems could be the dilute, old, and cold clouds of atomic hydrogen, which are observed in between spiral arms of our galaxy; see [16, 17] and references therein.

[7] Spherical 2D models of neutrino-driven heating of the stellar plasma around the nascent neutron star do not generate the observed supernova explosions [41, 42]. Although explosions may arise from hydrodynamical instabilities induced by medium anisotropies [43], a possibility for a more efficient neutrino heating of the stellar plasma may be an enhanced Fermi coupling $G_{\rm F}$ as compared to the zero-temperature and density value.

[8] By 'induce' we mean that the interaction between magnetic and electric charges necessarily is highly nonlocal [45–48].

underlying gauge dynamics with nonlocal interactions subject to $\mathrm{SU(2)_{CMB}}$, $\mathrm{SU(2)}_e$, $\mathrm{SU(2)}_\mu$, $\cdots$ One of the advantages of the latter description is, however, a deeper grasp of the particle–wave duality of the photon. That is, let γ be on-shell, thus propagating as a wave over large distances. Whenever γ approaches a charged lepton, it rotates into the associated massive Z_0, Ω_0, $\cdots$ excitation to interact with the charge. This process changes the wave into a massive particle transferring its momentum to the lepton in the subsequent collision.

4 Cosmological evolution from $z_{\mathrm{dec}} = 1089$ to $z = 0$

We consider a spatially flat Universe whose expansion is sourced by baryonic and dark, pressureless matter (M), a homogeneous axion field ϕ and $\mathrm{SU(2)_{CMB}}$ Yang–Mills thermodynamics. The evolution of the scale parameter $a = a(t)$ is determined by the Friedman equation:

$$H(t)^2 = (\dot{a}/a)^2 = \frac{8\pi}{3} G\left(\rho_{\mathrm{M}} + \rho_\phi + \rho_{\mathrm{CMB}}\right), \qquad (11)$$

where $G \equiv \frac{1}{M_{\mathrm{P}}^2}$ and $M_{\mathrm{P}} \equiv 1.2209 \times 10^{19}$ GeV. We are only interested in the evolution after CMB decoupling, i.e. for $z \leq z_{\mathrm{dec}} = 1089$. Within this range, the contribution of neutrinos can be neglected. Each of the contributions to the right-hand side of (11) is associated with separately conserved cosmological fluids as long as $z \geq 0$:

$$\mathrm{d}\left(\rho_i a^3\right) = -p_i\, \mathrm{d}(a^3), \quad (i = M, \phi, \mathrm{CMB}). \qquad (12)$$

Since $p_{\mathrm{M}} = 0$, we have $\rho_{\mathrm{M}}(a) = \rho_{\mathrm{M}}(a_0) \cdot (a_0/a)^3$, where t_0 is the present age of the Universe (to be calculated) and $a_0 \equiv a(t_0)$. In terms of the critical density $\rho_{\mathrm{c}} = 3\,H(t_0)^2/8\pi G = 4.08 \times 10^{11}\ \mathrm{eV}^4$, the measured matter contribution reads [49, 50]

$$\Omega_{\mathrm{M}} = \frac{\rho_{\mathrm{M}}}{\rho_{\mathrm{c}}}(a_0) = \Omega_{\text{Dark-Matter}} + \Omega_{\mathrm{Baryon}} = 0.27 \pm 0.04\,. \qquad (13)$$

By virtue of (12) (for the equation of state $p_{\mathrm{CMB}} = p_{\mathrm{CMB}}(\rho_{\mathrm{CMB}})$ see the appendix) the dependence $\rho_{\mathrm{CMB}} = \rho_{\mathrm{CMB}}(a)$ is calculated numerically. Notice, however, that at $t = t_{\mathrm{dec}}$, the contribution of $\mathrm{SU(2)_{CMB}}$ to the critical energy density is about 10% and decreases very rapidly for $t > t_{\mathrm{dec}}$. Although not directly affecting the evolution of the Universe, the presence of $\mathrm{SU(2)_{CMB}}$ is imprinted on the potential; for a Planck-scale axion (1) rewrite this potential as follows:

$$V(\phi) = (\lambda\Lambda_{\mathrm{CMB}})^4\left[1 - \cos\left(\frac{\phi}{F}\right)\right]. \qquad (14)$$

The dimensionless quantity λ parameterizes the uncertainty in the coupling of the topological defects of $\mathrm{SU(2)_{CMB}}$ to the axion. The value of λ is expected to lie within $O(10^{-1})$ to $O(10^1)$ [7, 8]. In our calculation, we adjust λ such that today's measured value of the dark-energy density is reproduced [49, 50]:

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{\mathrm{c}}}(a_0) = 1 - \Omega_{\mathrm{M}} = 0.73 \pm 0.04\,. \qquad (15)$$

The axion energy density ρ_ϕ and the pressure p_ϕ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \qquad (16)$$

From (12) and (16) the equation of motion for ϕ follows:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \qquad (17)$$

where $V' \equiv \mathrm{d}V/\mathrm{d}\phi$. The term $3H\dot{\phi}$ represents the cosmological 'friction'.

The origin of the field ϕ is the axial anomaly starting to be operative before inflation. In [1] it was concluded that the CMB-constraints on ϕ-induced adiabatic density perturbations would be such that the inflationary Hubble parameter is smaller than 10^{13} GeV. This entails that the scale F be larger than 10^{18} GeV $\simeq 0.1\,M_{\mathrm{P}}$. Moreover, a quantum field theoretic description in $(3+1)$ dimensions, which underlies the axial anomaly, likely is meaningful only below the Planck mass. Thus it is natural to suppose that $F \sim M_{\mathrm{P}}$.

The classical field ϕ, representing a condensate of axion particles being generated at $T \sim M_{\mathrm{P}}$, is surely fixed to its starting value $\phi_{\mathrm{in}} \sim F$ all the way down to CMB decoupling because of the large cosmological 'friction'. This implies the following initial conditions at decoupling:

$$\phi_{\mathrm{in}} = \phi(t_{\mathrm{dec}}) \sim F\,, \quad \dot{\phi}(t_{\mathrm{dec}}) = 0\,. \qquad (18)$$

We first consider $0 \leq \phi_{\mathrm{in}} \leq \pi\frac{F}{2}$, i.e. a range for which the curvature of the potential is positive. Let us now discuss the conditions under which the axion field behaves like a cosmological constant at present, that is, ϕ did not roll down its potential until now. This happens if[9]

$$3H(t_0) \gg 2m_\phi\,, \qquad (19)$$

where $m_\phi \equiv (\lambda\Lambda_{\mathrm{CMB}})^2/F$.

By using (11) and $V(\phi) \simeq \frac{1}{2}\, m_\phi^2\phi^2$ and neglecting the small direct contribution of $\mathrm{SU(2)_{CMB}}$, we have

$$H(t_0)^2 = \frac{4}{3}\pi G m_\phi^2 \phi_{\mathrm{in}}^2\left(1 + \frac{\Omega_{\mathrm{M}}}{\Omega_\phi}\right). \qquad (20)$$

Rewriting the condition (19) by using (20), we derive

$$\frac{\phi_{\mathrm{in}}}{M_{\mathrm{P}}} \gg \frac{1}{\sqrt{3\pi(1 + \Omega_{\mathrm{M}}/\Omega_\phi)}} \simeq 0.278\,. \qquad (21)$$

Even for $\phi_{\mathrm{in}}/M_{\mathrm{P}} \gtrsim 0.278$ slowly rolling solutions compatible with today's dark energy are numerically found; see

[9] $H(t)$ is a monotonically decreasing function; that is, if the condition $3H(t_0) \gg 2m_\phi$ is satisfied at t_0, it also holds for $t_{\mathrm{dec}} \leq t \leq t_0$.

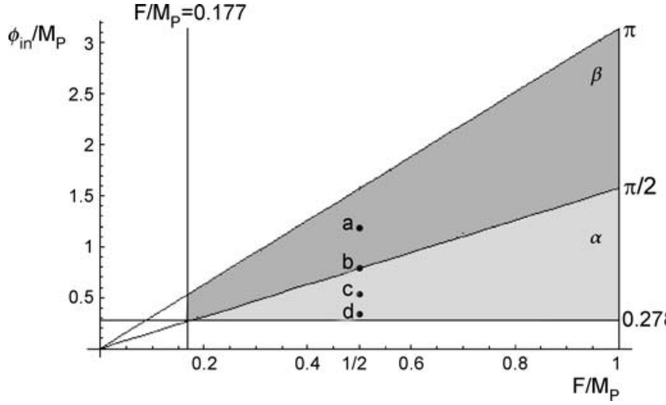

Fig. 3. Admissible range of the quantities $\phi_{\rm in}/M_{\rm P}$ and $F/M_{\rm P}$ for dark-energy-like axion-field solutions today. The *triangular area* α (β) corresponds to $\phi_{\rm in}/M_{\rm P}$ being below (above) the inflection point of $V(\phi)$. The *horizontal line* $\phi_{\rm in}/M_{\rm P} = 0.278$ indicates a rapid cross-over from slowly rolling to oscillating solutions

the discussion below. For $\phi_{\rm in}/M_{\rm P} \lesssim 0.278$, the parameter λ needs to assume unnaturally large values for the axion to generate today's value of the dark-energy density. Moreover, the axion would undergo many oscillations until today and thus would behave more like pressureless matter than dark energy.

For $0 \le \phi_{\rm in} \le \pi\frac{F}{2}$ to be meaningful when compared to the constraint of (21), one needs $F/M_{\rm P} > 0.177$. This is close to the lower bound $F/M_{\rm P} > 0.1 M_{\rm P}$ arising from the consideration on adiabatic density perturbations in [1]. In Fig. 3 admissible ranges for the initial conditions at $t = t_{\rm dec}$ are shown. The triangular area α represents the allowed parameter range for a slowly rolling field at present. The horizontal line $\phi_{\rm in}/M_{\rm P} = 0.278$ indicates a rapid crossover from dark-energy-like (above) to oscillating (below) solutions. The allowed range is enlarged by including the trapezoidal area β, corresponding to a negative curvature of the potential.[10] Notice that for $\pi F/2 \le \phi_{\rm in} \le \pi F$ there are slowly rolling solutions with the needed amount of present dark energy also for $F/M_{\rm P} < 0.177$. However, for a decreasing value of $F/M_{\rm P}$ we observe that $\phi_{\rm in}$ needs to be closer to the maximum πF, which is somewhat of a fine-tuned situation [5]. We thus pick out representative initial conditions as depicted in Fig. 3.

In Table 1 we present our numerical results obtained for the initial values corresponding to the points (a), (b), (c) and (d) in Fig. 3. The values of the following quantities are determined: λ such that $\Omega_\phi = 0.73$ at present, the present age t_0 of the Universe, the present Zeldovich parameter for the axion fluid alone, $w_\phi(t_0) = (p_\phi/\rho_\phi)_{t=t_0}$ (see (16)), and for the entire Universe, $w_{\rm tot}(t_0) = (p_{\rm tot}/\rho_{\rm tot})_{t=t_0}$, and the value of the redshift $z_{\rm acc}$ corresponding to the transition between decelerated and accelerated expansion.

For the set of initial values (a), (b), and (c) the axion field does not roll until t_0, as indicated by the quantity $w_\phi(t_0) \simeq -1$. For point (d) $\phi_{\rm in}/M_{\rm P}$ is just above the

[10] If the field does not roll at $\phi_{\rm in} = \pi F/2$ (inflexion point), then it also does not roll for $\pi F/2 \le \phi_{\rm in} \le \pi F$.

Table 1. The values of selected cosmological parameters obtained for variable initial values at CMB decoupling, keeping $F/M_{\rm P} = 0.5$ fixed

points in Fig. 3	$\phi_{\rm in}/M_{\rm P}$	λ	t_0 (Gy)	$w_\phi(t_0)$	$w_{\rm tot}(t_0)$	$z_{\rm acc}$
(a)	$3\pi/4$	22.15	13.65	−0.97	−0.71	0.76
(b)	$\pi/4$	22.15	13.65	−0.97	−0.71	0.76
(c)	$\pi/6$	26.96	13.56	−0.91	−0.66	0.79
(d)	0.328	37.27	13.08	−0.61	−0.44	0.92

threshold in (21) causing the field ϕ to roll at present: $w_\phi(t_0) = -0.61$. According to [49, 50], $w_\phi(t_0) = -0.61$ is already inconsistent with observation ($w_\phi(t_0) < -0.78$ at 95% C.L.). Decreasing $\phi_{\rm in}/M_{\rm P}$ further one rapidly runs into the regime $w_{\rm tot}(t_0) > -1/3$, where the present Universe does not accelerate. The values of $z_{\rm acc}$ obtained for (a), (b), and (c) are in approximate agreement with $z_{\rm acc} = 0.75$ obtained for a standard ΛCDM model. Moving $\frac{F}{M_{\rm P}}$ within the allowed range $\alpha \cup \beta$ at fixed values of $\phi_{\rm in}/M_{\rm P}$, see Fig. 3, the values of the cosmological parameters in Table 1 are almost unaffected.

Due to the dynamical nature of dark energy in our model, the Universe will not run into pure de Sitter expansion in the future as it does for the ΛCDM model, but rather epochs of accelerated and decelerated expansions will alternate: $z_{\rm acc}$ corresponds to the first of many more future turning points ($\ddot{a} = 0$). This, however, presumes that the axion–SU(2)$_{\rm CMB}$ coupling will remain unaffected by the future evolution.

5 A massive photon in the future

Here we consider the future evolution up to the point where SU(2)$_{\rm CMB}$ undergoes the transition to its preconfining phase. For simplicity we assume that the present age of the Universe t_0 is given by the time when $T_0 = T_{\rm CMB}$ is first reached. Because of the discontinuity of the energy density (Fig. 2)

$$\frac{\rho_{\rm c,M}}{T^4_{\rm CMB}} - \frac{\rho_{\rm c,E}}{T^4_{\rm CMB}} = \frac{4}{3}\frac{\pi^2}{30}\,, \tag{22}$$

the system cannot jump into the preconfining (magnetic) phase where the photon possesses a mass and thus an additional polarization because of condensed monopoles in the ground state. The energy gap $\frac{4}{3}\frac{\pi^2}{30}$ is the sum of the energy gap of the photon gas ($= \frac{\pi^2}{30}$) and the ground state ($= \frac{1}{3}\frac{\pi^2}{30}$); we refer to the appendix for the details. Therefore, the system remains in the deconfining (electric) phase in a supercooled state (with its ground state still being an ensemble of interacting calorons instead of monopoles) so long as the energy density of the electric phase $\rho_{\rm E}$ is smaller than the energy density of the magnetic phase $\rho_{\rm M}$ ($\rho_{\rm E} < \rho_{\rm M}$). At a certain value of the temperature, $\lambda_{*,{\rm E}} = 2\pi T_*/\Lambda_{\rm E} < \lambda_{\rm c,E} = 2\pi T_{\rm c,E}/\Lambda_{\rm E}$, equality $\rho_{\rm E} = \rho_{\rm M}$ takes place. At this point the condensation

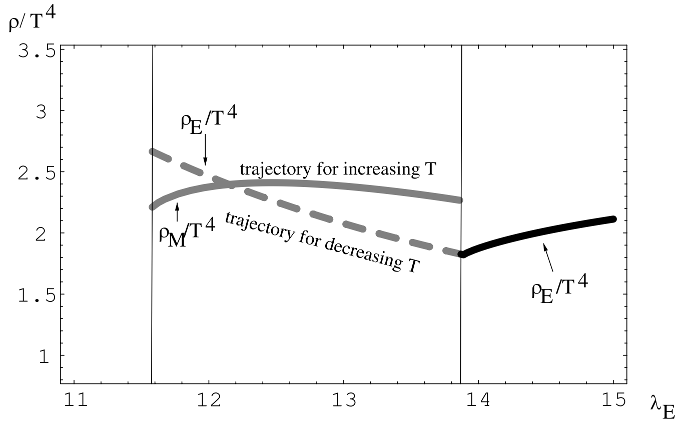

Fig. 4. The situation for the (dimensionless) energy density in the critical region: the *dashed line* represents the continuation of the energy density of the deconfining phase (*solid black line*) for $T < T_{\mathrm{c,E}}$ (supercooled state, realized for decreasing temperature, $m_\gamma = 0$). The *solid grey line* depicts the energy density in the preconfining phase (realized for increasing temperature, $m_\gamma > 0$). At the intersection point, $\lambda_{\mathrm{E}} = \lambda_{*,\mathrm{E}}$, a phase transition from the supercooled deconfining to the preconfining dynamics occurs

of monopoles occurs and the photon becomes massive (for $\lambda_{*,\mathrm{E}} \leq \lambda_{\mathrm{E}} \leq \lambda_{\mathrm{c,E}}$, monopoles are not sufficiently liberated by the associated large-holonomy calorons to facilitate unlimited mobility). The situation is depicted in Fig. 4, where the dashed line represents the continuation of the (dimensionless) energy density $\rho_{\mathrm{E}}/\Lambda_{\mathrm{E}}^4$ for $\lambda_{\mathrm{E}} < \lambda_{\mathrm{c,E}}$. The corresponding analytical expressions are given in the appendix [13]. The intersection $\rho_{\mathrm{E}}/T^4 = \rho_{\mathrm{M}}/T^4$ occurs at $\lambda_{*,\mathrm{E}} = 12.15 < \lambda_{\mathrm{c,E}} = 13.87$ (i.e. $T_*/T_{\mathrm{c,E}} = 0.88$) (see Fig. 4; for technical details see the appendix). Driven by cosmological expansion, which essentially is sourced by dark matter and the axion field, the $\mathrm{SU(2)_{CMB}}$ thermodynamics evolves into a supercooled state (deconfining phase) according to (12) ($i = \mathrm{CMB}$) down to the point where the density ρ_* (i.e. the transition temperature T_*) is reached. The numerical result for the scale factor at $T = T_*$ is $a(t_*)/a_0 = 1.17$. At this point the photon acquires a Meissner mass. Notice that according to (2) the anomaly-mediated decay width $\Gamma_{\phi\to 2\gamma}$ of the axion into two photons is much smaller than the present Hubble parameter H_0:

$$\Gamma_{\phi\to 2\gamma} < \left(\frac{m_\phi}{F}\right)^2 m_\phi \sim 10^{-155}\,\mathrm{eV} \lll H_0 \sim 10^{-33}\,\mathrm{eV}\,. \tag{23}$$

Thus it is justified to treat the axion as a coherent field for any practical purpose and to consider the axion and the $\mathrm{SU(2)_{CMB}}$ fluids to be separately conserved as in (12).

The numerical value of the time interval $\Delta t_{m_\gamma=0} = t_{*,\mathrm{E}} - t_0$ follows from future cosmology according to (11). For the sets of initial values (a)–(d) in Table 1 we obtain the following numbers:

$$\text{(a) } \Delta t_{m_\gamma=0} = 2.20\,\mathrm{Gyr}\,, \quad \text{(b) } \Delta t_{m_\gamma=0} = 2.20\,\mathrm{Gyr}\,,$$
$$\text{(c) } \Delta t_{m_\gamma=0} = 2.22\,\mathrm{Gyr}\,, \quad \text{(d) } \Delta t_{m_\gamma=0} = 2.29\,\mathrm{Gyr}\,. \tag{24}$$

The value $\Delta t_{m_\gamma=0} \simeq 2.2\,\mathrm{Gyr}$ depends only weakly on the chosen parameter set. The error in determining the quantity $\Delta t_{m_\gamma=0}$ is dominated by the observational uncertainty for the present Hubble parameter H_0. According to [49, 50], we have $\frac{\delta H(t_0)}{H(t_0)} \sim \pm 0.056$. We have run our simulations with the upper (lower) limit for the error range. This generates a decrease (increase) for $\Delta t_{m_\gamma=0}$ of about 0.15 Gyr.

Throughout the work we assumed that the temperature $T_{\mathrm{c,E}} = T_{\mathrm{CMB}}$ is reached today. However, the photon is massless and unscreened in the entire range $T_* \leq T \leq T_{\mathrm{c,E}}$. Therefore the present CMB-temperature could also be below $T_{\mathrm{c,E}}$. As a consequence, the quantity $\Delta t_{m_\gamma=0}$ represents an upper bound for the time interval between the present and the occurrence of the phase transition.

The existence of extra-galactic magnetic fields [52, 53] could indeed signal the onset of a superconducting vacuum. Possibly, a quantitative analysis would rely on tunneling effects connecting the two trajectories in Fig. 4. Such an unconventional interpretation of extra-galactic magnetic fields needs future investigation.

6 Summary and conclusions

We have elaborated the idea that the present density of dark energy arises from an ultralight axion field with a Peccei–Quinn scale comparable to M_{P} [1]. More precisely, we have linked the normalization μ^4 of the axion potential in (1) to the existence of an SU(2) Yang–Mills theory of a scale $\Lambda_{\mathrm{CMB}} \sim \mu$, comparable to the temperature T_{CMB} of the present cosmic microwave background. Such an assertion has its justification in nonperturbative results obtained recently for SU(2) Yang–Mills thermodynamics [13]. As a result, we have obtained an upper bound $\Delta t_{m_\gamma=0} \simeq 2\,\mathrm{Gyr}$ for the length of the time interval from the present to the phase transition where the photon acquires a Meissner mass.

Throughout our work we have assumed a cold dark matter component $\rho_{\mathrm{M}} = 0.23\rho_{\mathrm{c}}$ of unknown origin at present. (A possibility would be that ρ_{M} arises due to the decay of one or more oscillating, coherent axion fields ϕ_i with $F_i \ll M_{\mathrm{P}}$ into their particles at earlier epochs.) A more unified but also more speculative picture would arise if today's rolling axion field would describe both dark matter and dark energy; see [54] and references therein. On the one hand, according to our simulations (performed with a canonical kinetic term) such a scenario would imply an age of the Universe of about 20 Gyr as opposed to 13.7 Gyr with conventional cold dark matter. Also one would obtain $z_{\mathrm{acc}} \sim 3$, as opposed to $z_{\mathrm{acc}} \sim 0.75$, possibly endangering structure formation. On the other hand, structure formation and the flattening of the rotation curves of galaxies would need an explanation in terms of ripples and lumps of a coherent axion field [55]. Moreover, the relation between luminosity distance and redshift as observed from SNe Ia standard candles would have to be postdicted with a pressureless contribution to the Hubble parameter that acquired nominal strength only very recently. The future will tell (gravitational lensing signatures for galaxies, theoretical results on the stability of the system axion-lump plus baryonic matter plus gravity) whether such a possibility is viable.

For completeness we have investigated how the latter scenario affects our estimate $\Delta t_{m_\gamma=0}$. By defining the quantity η through $\eta=-p_\phi$ and $\rho_\phi=\dot{\phi}^2+\eta$, the axion fluid can be split into a component with $\rho_\Lambda=\eta$ (with $w_\Lambda=-1$) and a component $\rho_{\mathrm{DM}}=\dot{\phi}^2$ (with $w_{\mathrm{DM}}=0$). Notice that the components so defined are not separately conserved. The task is to uniquely fix ϕ_{in} and λ in (14) such that $\Omega_\phi=0.96$ today (with $\Omega_{\mathrm{Baryon}}=0.04$)) and such that $\Omega_\Lambda=0.73$ and $\Omega_{\mathrm{DM}}=0.23$. Using $\frac{F}{M_{\mathrm{P}}}=0.5$, we obtain $\frac{\phi_{\mathrm{in}}}{M_{\mathrm{P}}}=0.53$ and $\lambda=31.9$. This yields $\Delta t_{m_\gamma=0}=2.21\,\mathrm{Gyr}$. Thus our estimate $\Delta t_{m_\gamma=0}$ is rather model independent.

Finally, let us make a few comments concerning future activity. The postulate $\mathrm{SU(2)_{CMB}} \overset{\text{today}}{=} \mathrm{U(1)_Y}$ entails consequences for the CMB map of fluctuations in temperature and in electric/magnetic field polarization at large angles [14, 16, 17]. To analyze these effects more quantitatively one needs precise information on the underlying cosmology; this basic step was addressed in the present work.

For the viability of the postulate $\mathrm{SU(2)_{CMB}} \overset{\text{today}}{=} \mathrm{U(1)_Y}$, an alternative interpretation of electroweak SM physics in terms of underlying, nonperturbative and pure Yang–Mills dynamics is necessary. Although we have checked a few experimental benchmarks on this scenario, further theoretical work surely is needed.

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Appendix : Expressions for the energy density and the pressure

We start from the effective Lagrangians in the deconfining and preconfining phases as described in [13] and we briefly derive in a self-contained way the corresponding one-loop thermodynamical quantities. Higher-order loop corrections turn out to be of the order of 0.1% [14]; thus they are irrelevant for our cosmological application.

A.1 Deconfining (electric) phase

The deconfining (electric) phase occurs for $T>T_{\mathrm{c,E}}=\Lambda_{\mathrm{E}}\lambda_{\mathrm{c,E}}/2\pi$, $\lambda_{\mathrm{c,E}}=13.87$, where Λ_{E} is the Yang–Mills scale in the electric phase.

The effective Lagrangian for the description of SU(2) Yang–Mills thermodynamics in the deconfining phase and in the unitary gauge reads [13]

$$\mathcal{L}^{\mathrm{u.g.}}_{\text{dec-eff}}=\frac{1}{4}\left(G^{a,\mu\nu}_{\mathrm{E}}[a_\mu]\right)^2+2e(T)^2\left|\varphi_{\mathrm{E}}\right|^2 \times\left(\left(a_\mu^{(1)}\right)^2+\left(a_\mu^{(2)}\right)^2\right)+\frac{2\Lambda_{\mathrm{E}}^6}{\left|\varphi_{\mathrm{E}}\right|^2}, \quad \text{(A.1)}$$

where $G^a_{\mathrm{E},\mu\nu}=\partial_\mu(a^a_\nu)-\partial_\nu(a^a_\mu)+e\varepsilon^{abc}a^b_\mu a^c_\nu$ is the SU(2) stress-energy tensor for the topologically trivial fluctuations a^a_μ (with effective coupling $e=e(T)$) and the adjoint scalar background field φ_{E} embodies the spatial coarse-graining of caloron and anticaloron field configurations (see [13] for a microscopic derivation, see [56] for a macroscopic one). The quantum fluctuations $a_\mu^{(1,2)}$, in our work identified by $V^\pm$, are massive, while the gauge mode $a_\mu^{(3)}$, here the photon, stays massless (spontaneous symmetry breaking $\mathrm{SU(2)}\to\mathrm{U(1)}$). The mass of $V^\pm$ reads explicitly (see (A.1))

$$m=m_{V^+}=m_{V^-}=2e(T)\left|\varphi_{\mathrm{E}}\right|\,;\quad \left|\varphi_{\mathrm{E}}\right|=\sqrt{\frac{\Lambda_{\mathrm{E}}^3}{2\pi T}}\,. \quad \text{(A.2)}$$

At this stage the effective running coupling $e=e(T)$ is not yet known. Its behavior is determined by imposing thermodynamical self-consistency; see below.

The energy density and the pressure are the sums of three terms,

$$\begin{aligned}\rho_{\mathrm{E}}&=\rho_{\mathrm{E},\gamma}+\rho_{\mathrm{E},V^\pm}+\rho_{\mathrm{E,gs}}\,,\\ p_{\mathrm{E}}&=p_{\mathrm{E},\gamma}+p_{\mathrm{E},V^\pm}+p_{\mathrm{E,gs}}\,,\end{aligned} \quad \text{(A.3)}$$

corresponding to the contributions of the massless gauge mode γ, the two massive gauge modes $V^\pm$ and the ground state, respectively. The one-loop expressions are easily obtained from the effective Lagrangian (A.1):

$$\begin{aligned}\rho_{\mathrm{E},\gamma}&=2\frac{\pi^2}{30}T^4\,,\\ \rho_{\mathrm{E},V^\pm}&=6\int_0^\infty\frac{k^2\,\mathrm{d}k}{2\pi^2}\frac{\sqrt{m^2+k^2}}{\exp\left(\frac{\sqrt{m^2+k^2}}{T}\right)-1}\,,\\ \rho_{\mathrm{E,gs}}&=4\pi\Lambda_{\mathrm{E}}^3T\,,\end{aligned} \quad \text{(A.4)}$$

$$\begin{aligned}p_{\mathrm{E},\gamma}&=2\frac{\pi^2}{90}T^4\,,\\ p_{\mathrm{E},V^\pm}&=-6T\int_0^\infty\frac{k^2\,\mathrm{d}k}{2\pi^2}\ln\left(1-\mathrm{e}^{-\frac{\sqrt{m^2+k^2}}{T}}\right)\,,\\ p_{\mathrm{E,gs}}&=-\rho_{\mathrm{E,gs}}\,.\end{aligned} \quad \text{(A.5)}$$

We first rewrite the system in terms of dimensionless quantities:

$$\begin{aligned}&\bar{\rho}_{\mathrm{E}}=\frac{\rho_{\mathrm{E}}}{T^4}\,,\quad \bar{p}_{\mathrm{E}}=\frac{p_{\mathrm{E}}}{T^4}\,,\quad \lambda=\lambda_{\mathrm{E}}=\frac{2\pi T}{\Lambda_{\mathrm{E}}}\,,\\ &a(\lambda)=\frac{m(T)}{T}=2\frac{e(T)}{T}\left|\varphi_{\mathrm{E}}\right|\,,\end{aligned} \quad \text{(A.6)}$$

where the function $a=a(\lambda)$ has been introduced for later use.

The dimensionless density and pressure, expressed as functions of the dimensionless temperature λ, read

$$
\begin{aligned}
\bar{\rho}_{\mathrm{E},\gamma} &= 2\frac{\pi^2}{30}\,,\\
\bar{\rho}_{\mathrm{E},V^\pm} &= \frac{3}{\pi^2}\int_0^\infty \mathrm{d}x\,\frac{x^2\sqrt{x^2+a^2}}{\mathrm{e}^{\sqrt{x^2+a^2}}-1}\,,\\
\bar{\rho}_{\mathrm{E,gs}} &= \frac{2(2\pi)^4}{\lambda^3}\,,
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
\bar{p}_{\mathrm{E},\gamma} &= 2\frac{\pi^2}{90}\,,\\
\bar{p}_{\mathrm{E},V^\pm} &= -\frac{3}{\pi^2}\int_0^\infty x^2\,\mathrm{d}x\ln\left(1-\mathrm{e}^{-\sqrt{x^2+a^2}}\right)\,,\\
\bar{p}_{\mathrm{E,gs}} &= -\bar{\rho}_{\mathrm{E,gs}}\,.
\end{aligned} \tag{A.8}
$$

We impose the validity of the thermodynamical Legendre transformation:

$$
\rho = T\frac{\mathrm{d}P}{\mathrm{d}T} - P \iff \bar{\rho} = \lambda\frac{\mathrm{d}\bar{p}}{\mathrm{d}\lambda} + 3\bar{p} \tag{A.9}
$$

(expressed with both dimensional and dimensionless functions). By substituting the expressions (A.7) and (A.8) into (A.9), we determine the following differential equation for $a = a(\lambda)$:

$$
1 = -\frac{6\lambda^3}{(2\pi)^6}\left(\lambda\frac{\mathrm{d}a}{\mathrm{d}\lambda} + a\right)aD(a)\,, \tag{A.10}
$$

$$
D(a) = \int_0^\infty \mathrm{d}x\,\frac{x^2}{\sqrt{x^2+a^2}}\,\frac{1}{\mathrm{e}^{\sqrt{x^2+a^2}}-1}\,,\quad a(\lambda_{\mathrm{in}}) = 0\,. \tag{A.11}
$$

For a sufficiently large initial value of λ_{in}, the solution for $a(\lambda)$ is independent on λ_{in}: a low-temperature attractor with a logarithmic pole at $\lambda_{\mathrm{c,E}} = 13.87$ is seen (infinite mass for $V^\pm$ leading to their thermodynamical decoupling). The effective coupling is given by $e = e(\lambda) = a(\lambda)\lambda^{3/2}/4\pi$ and shows a plateau at $e \sim 8.89$; see Fig. 1.

Once the function $a = a(\lambda)$ is determined, density and pressure in the deconfining phase are numerically obtained and are plotted in Fig. 2.

A.2 Preconfining (magnetic) phase

The preconfining (magnetic) phase occurs for $11.57\Lambda_{\mathrm{E}}/2\pi = T_{\mathrm{c,M}} \le T \le T_{\mathrm{c,E}} = \Lambda_{\mathrm{E}}\lambda_{\mathrm{c,E}}/2\pi$.

The effective Lagrangian for the description of SU(2) Yang–Mills thermodynamics in the preconfining phase and in the unitary gauge reads [13]

$$
\mathcal{L}^{\mathrm{u.g.}}_{\text{prec-eff}} = \frac{1}{4}(F_{\mathrm{E}}^{\mu\nu})^2 + \frac{1}{2}g^2\left|\varphi_{\mathrm{M}}\right|^2\left(a_\mu^{(3)}\right)^2 + \frac{2\Lambda_{\mathrm{M}}^6}{\left|\varphi_{\mathrm{M}}\right|^2}\,, \tag{A.12}
$$

where $F_{\mathrm{E}}^{\mu\nu} = \partial_\mu a_\nu^{(3)} - \partial_\nu a_\mu^{(3)}$ is the (dual) abelian field strength tensor, the complex scalar field φ_{M} describes the condensate of magnetic monopoles, and Λ_M the Yang–Mills scale in the magnetic phase. The remaining U(1) symmetry of the deconfining phase is broken. The photon acquires a temperature-dependent mass:

$$
m_\gamma = g\left|\varphi_{\mathrm{M}}\right| = bT\,,\quad \left|\varphi_{\mathrm{M}}\right| = \sqrt{\frac{\Lambda_{\mathrm{M}}^3}{2\pi T}}\,, \tag{A.13}
$$

where the function $b \equiv b(\lambda)$ is introduced for later use. The effective coupling $g \equiv g(\lambda)$ is not yet known. As before, thermodynamical self-consistency will determine its behavior.

The energy density and the pressure are now the sums of two terms referring to the photon and to the ground state respectively:

$$
\begin{aligned}
\rho_{\mathrm{M}} &= \rho_{\mathrm{M},\gamma} + \rho_{\mathrm{M,gs}}\,,\\
p_{\mathrm{M}} &= p_{\mathrm{M},\gamma} + p_{\mathrm{M,gs}}\,,
\end{aligned} \tag{A.14}
$$

with

$$
\begin{aligned}
\rho_{\mathrm{M},\gamma} &= 3\int_0^\infty \frac{k^2\,\mathrm{d}k}{2\pi^2}\,\frac{\sqrt{m_\gamma^2+k^2}}{\exp\left(\frac{\sqrt{m_\gamma^2+k^2}}{T}\right)-1}\,,\\
\rho_{\mathrm{M,gs}} &= 4\pi\Lambda_{\mathrm{M}}^3 T\,,
\end{aligned} \tag{A.15}
$$

$$
\begin{aligned}
p_{\mathrm{M},\gamma} &= -3T\int_0^\infty \frac{k^2\,\mathrm{d}k}{2\pi^2}\ln\left(1-\mathrm{e}^{-\frac{\sqrt{m_\gamma^2+k^2}}{T}}\right)\,,\\
p_{\mathrm{M,gs}} &= -\rho_{\mathrm{M,gs}}\,.
\end{aligned} \tag{A.16}
$$

The corresponding dimensionless quantities are expressed in terms of $\lambda = 2\pi T/\Lambda_{\mathrm{E}}$ and read

$$
\begin{aligned}
\bar{\rho}_{\mathrm{M},\gamma} &= \frac{3}{2\pi^2}\int_0^\infty \mathrm{d}x\,\frac{x^2\sqrt{x^2+b^2}}{\mathrm{e}^{\sqrt{x^2+b^2}}-1}\,,\\
\bar{\rho}_{\mathrm{M,gs}} &= \left(\frac{\Lambda_{\mathrm{M}}}{\Lambda_{\mathrm{E}}}\right)^3\frac{2(2\pi)^4}{\lambda^3}\,,
\end{aligned} \tag{A.17}
$$

$$
\begin{aligned}
\bar{p}_{\mathrm{M},\gamma} &= -\frac{3}{2\pi^2}\int_0^\infty x^2\,\mathrm{d}x\ln\left(1-\mathrm{e}^{-\sqrt{x^2+b^2}}\right)\,,\\
\bar{p}_{\mathrm{M,gs}} &= -\bar{\rho}_{\mathrm{M,gs}}\,.
\end{aligned} \tag{A.18}
$$

The ratio $\Lambda_{\mathrm{M}}/\Lambda_{\mathrm{E}}$ is determined by imposing that the pressure and the photon mass be continuous functions across the phase transition at $\lambda_{\mathrm{c,E}} = 13.87$ (second-order phase transition [13]). A continuous photon mass requires $b(\lambda \to \lambda_{\mathrm{c,E}}^-) = 0$. Thus, by equating (A.8) and (A.18), we obtain the following matching condition:

$$
\left(\frac{\Lambda_{\mathrm{M}}}{\Lambda_{\mathrm{E}}}\right)^3 = 1 + \frac{\lambda_{\mathrm{c,E}}^3}{720(2\pi)^2} = 1.09\,. \tag{A.19}
$$

As a consequence, the energy gap at the phase transition reads

$$
\Delta = \bar{\rho}_{\mathrm{M}}(\lambda_{\mathrm{c,E}}) - \bar{\rho}_{\mathrm{E}}(\lambda_{\mathrm{c,E}}) = \frac{4}{3}\frac{\pi^2}{30}\,. \tag{A.20}
$$

We require thermodynamical self-consistency in the magnetic phase: by substituting the expressions (A.17) and (A.18) into (A.9) we obtain the following differential equation for $b = b(\lambda)$:

$$1 = -\frac{3\lambda^3}{(2\pi)^6}\left(\frac{\Lambda_\mathrm{E}}{\Lambda_\mathrm{M}}\right)^3\left(\lambda\frac{\mathrm{d}b}{\mathrm{d}\lambda}+b\right)bD(b)\,,$$
$$b(\lambda=\lambda_{\mathrm{c,E}})=0\,. \tag{A.21}$$

The behavior of the magnetic coupling $g(\lambda) = b(\lambda)\lambda^{3/2}/4\pi$ is shown in Fig. 1. A pole at the critical value $T_{\mathrm{c,M}} = 11.57\Lambda_E/2\pi$ is encountered. At this temperature the photon mass diverges. For temperature smaller than $T_{\mathrm{c,M}}$, the system is in a completely confining phase.

A.3 Supercooled electric phase

The supercooled electric phase occurs for $\Lambda_\mathrm{E}\lambda_{*,\mathrm{E}}/2\pi = T_* \le T \le T_{\mathrm{c,E}} = \Lambda_\mathrm{E}\lambda_{\mathrm{c,E}}/2\pi$, $\lambda_{*,\mathrm{E}} = 12.15$.

The system, cooling down from the deconfining into the preconfining phase, stays in a supercooled deconfining phase by virtue of the energy gap. Energy density and pressure in the supercooled deconfining phase are given by

$$\bar\rho_\mathrm{E}^{\mathrm{s.c.}} = \bar\rho_{\mathrm{E},\gamma}^{\mathrm{s.c.}} + \bar\rho_{\mathrm{E,gs}}\,,$$
$$\bar p_\mathrm{E}^{\mathrm{s.c.}} = \bar p_{\mathrm{E},\gamma}^{\mathrm{s.c.}} + \bar p_{\mathrm{E,gs}}^{\mathrm{s.c.}}\,, \tag{A.22}$$

where

$$\bar\rho_{\mathrm{E},\gamma}^{\mathrm{s.c.}} = 2\frac{\pi^2}{30}\,,\quad \bar\rho_{\mathrm{E,gs}} = \frac{2(2\pi)^4}{\lambda^3}\,;$$
$$\bar p_{\mathrm{E},\gamma}^{\mathrm{s.c.}} = 2\frac{\pi^2}{90}\,,$$
$$\bar p_{\mathrm{E,gs}}^{\mathrm{s.c.}} = -\bar\rho_{\mathrm{E,gs}}\,. \tag{A.23}$$

The behavior of $\bar\rho_\mathrm{E}^{\mathrm{s.c.}}$ and $\bar\rho_\mathrm{M}$ is indicated in Fig. 4, where the corresponding transition value $\lambda_{*,\mathrm{E}}$ ($\bar\rho_\mathrm{E}^{s.c.}(\lambda_{*,\mathrm{E}}) = \bar\rho_\mathrm{M}(\lambda_{*,\mathrm{E}})$) is determined to be $\lambda_{*,\mathrm{E}} = 12.15$.

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